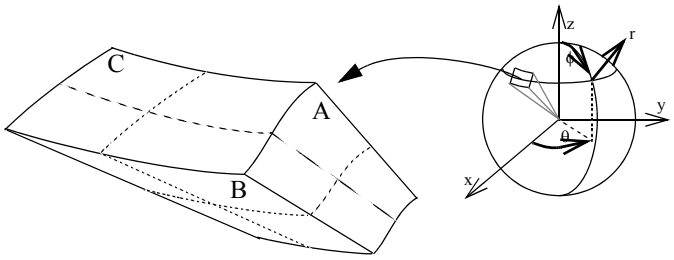
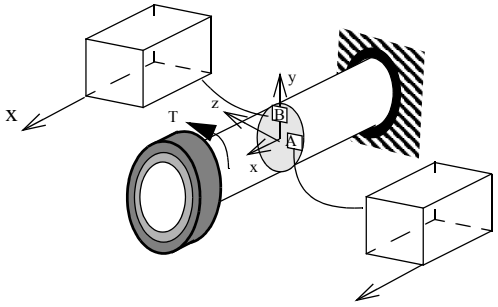
EXAM 1 MEEM 4150 Oct. 4th, 2011

1. (a) Show the non-zero stress components in the r, θ, and ϕ spherical coordinate system on the A,B, and C faces of the stress element shown.

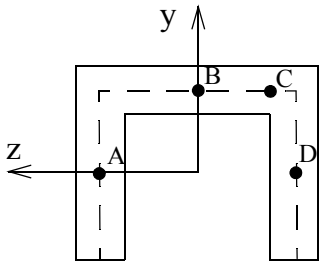
$$\begin{bmatrix} \sigma_{rr} = 0 & \tau_{r\theta} = -18\text{ksi} & \tau_{r\phi} = 0\text{ksi} \\ \tau_{\theta r} = -18\text{ksi} & \sigma_{\theta\theta} = 10\text{ksi(C)} & \tau_{\theta\phi} = 25\text{ksi} \\ \tau_{\phi r} = 0 & \tau_{\phi\theta} = 25\text{ksi} & \sigma_{\phi\phi} = 20\text{ksi(T)} \end{bmatrix}$$



(b) Show the direction of shear stress on the *four relevant* surfaces at points A and B on the given stress cubes. Circle the correct component and sign.



(c) Assuming a positive shear force V_y in bending (a) sketch the direction of the shear flow along the center-line on the thin cross-sections shown. (b) At points A, B, C, and D, determine if the stress component is τ_{xy} or τ_{xz} and if it is positive, negative or zero. Circle the appropriate answers



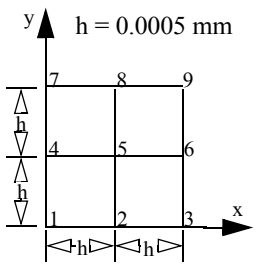
(d, e) The stresses at a point were found to be σ_{rr}= 100 MPa (T), $\sigma_{\theta\theta}$ = 200 MPa (T) and $\tau_{r\theta}$ = 140 MPa . The material has a modulus of elasticity of 80 GPa and Poisson's ratio of 0.25. Determine ε_{rr}

(d) assuming *Plane Stress*

(e) assuming *Plane Strain*

(f) The displacements u and v in the x and y directions respectively were measured by Moire' interferometry. Displacements of 9 points on the body and are as given below. Determine the shear strain γ_{xy} at point 8.

Point	u (μmm)	v (μmm)	Point	u (μmm)	v (μmm)
1	0.000	0.000	6	-0.080	0.240
2	-0.112	0.144	7	0.128	0.384
3	-0.128	0.256	8	-0.048	0.336
4	0.112	0.176	9	0.128	0.384
5	-0.032	0.224			

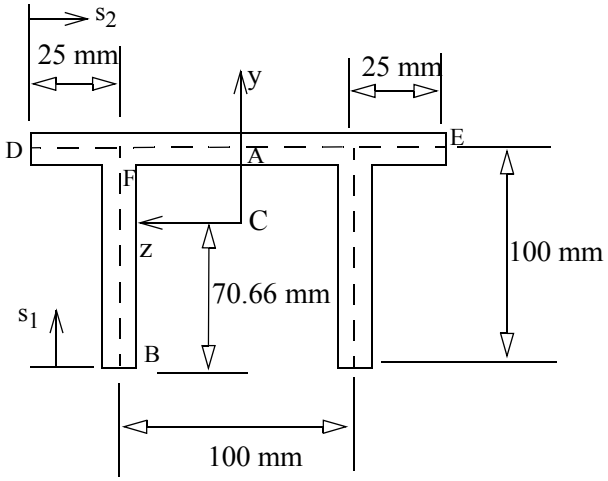


2. The principal stresses for the given stress matrix are $\sigma_1 = 28.10\ \text{ksi(T)}$; $\sigma_2 = 7.48\ \text{ksi(T)}$; and $\sigma_3 = 25.58\ \text{ksi(C)}$. (a) Determine the three stress invariants (I_1, I_2, I_3). (b) Determine the principal angles associated with the third principal stress 25.58 ksi (C) (c) Determine the stress σ_{xx}. (d) The material yield stress is 60 ksi in tension. Determine the factor of safety (K) at the point using octahedral shear stress theory. (e) The critical stress intensity factor for the material is $22\text{ksi}\sqrt{\text{in}}$, what is the critical crack length for the material.

$$\begin{bmatrix} \sigma_{xx} & -10 & 5 \\ -10 & 15 & 0 \\ 5 & 0 & -25 \end{bmatrix} \text{ksi}$$

Stress Matrix

3. A positive shear force of V_y=10 kN and a bending moment of M_z = 5 kN-m acts on the thin cross-sections shown (not drawn to scale). The cross-section has a uniform thickness of 10 mm and an area moment of inertia about the z-axis $I_{zz} = 3.7518(10^6)\text{mm}^4$ (a) Determine the shear flow along center line for segment BF and DF in terms of s_1 and s_2, respectively. (b) The principal stresses at point F on the cross-section. Point F is just below the flange. Report your answers with proper units.



Answers

1 (b) $(\tau_{xy})_A$ = positive; $(\tau_{xz})_B$ = positive; (c) $(\tau_{xy})_A$ positive; $(\tau_{xz})_B$ zero; $(\tau_{xz})_C$ positive; $(\tau_{xy})_D$ positive; (d) $\varepsilon_{\theta\theta} = 218.7\ \mu$ (e) $\varepsilon_{\theta\theta} = 1953.1\ \mu$; (f) γ_{xy} =- 32 μ rads

2. $I_1 = 10\ \text{ksi}$; $I_2 = -700\ \text{ksi}^2$; $I_3 = -5377\ \text{ksi}^3$; $\sigma_{xx} = 20\ \text{ksi (T)}$; $\theta_x = 83.38^o$; $\theta_y = 88.37^o$; $\theta_z = 173.2^o$; K = 1.27; $2a_{crit} = 0.39$ in.

3. $q_1 = -[13.3s_1^2 - 1883.4s_1]$ N/m; $q_2 = -782s_2$ N/m; $\sigma_1 = 1.03$ MPa (T) $\sigma_2 = 33.5$ MPa (C) $\sigma_3 = 0$